The text below is extracted from the following paper:

Large-eddy simulation of turbulent airflow over complex terrain
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CHARACTERISTICS OF THE RIAM-COMPACT®

Governing equations

We consider a three-dimensional airflow of incompressible and viscous fluid over complex terrain with characteristic length scales on the order of kilometers, so that the Coriolis force can be neglected. In a DNS, the dimensional governing equations consist of the continuity and Navier-Stokes equations, as follows:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (1),$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho_0}\frac{\partial p}{\partial x_i} + \frac{\mu}{\rho_0}\frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad (2),$$

where the subscripts i and j=1, 2, and 3 correspond to the streamwise (x), spanwise (y), and vertical (z) directions, respectively. In the above equations, u_i is the instantaneous velocity component in the i-direction, p is the instantaneous pressure, ρ_0 is the reference density, and μ is the viscosity coefficient. All the variables are non-dimensionalized by an appropriate velocity U_{in} and a length scale h, such as $u_i^* = u_i/U_{in}$ and $x_i^* = x_i/h$, resulted in the following dimensionless equations:

$$\frac{\partial u_i}{\partial x_i} = 0 \quad (3),$$

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j \partial x_j} \quad (4),$$

where Re ($= \rho_0 U_{in} h/\mu$) is the Reynolds number, and the asterisk is omitted.

In an LES, the flow variables are divided into a GS (Grid-Scale) part and a SGS (SubGrid-Scale) part by the filtering operation. The filtered continuity and Navier-Stokes equations written in non-dimensional form are given by

$$\frac{\partial \overline{u}_i}{\partial x_i} = 0 \quad (5),$$

$$\frac{\partial \overline{u}_i}{\partial t} + \overline{u}_j \frac{\partial \overline{u}_i}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} - \frac{\partial \tau_{ij}}{\partial x_j} + \frac{1}{Re}\frac{\partial^2 \overline{u}_i}{\partial x_j \partial x_j}$$

(6),

where $\overline{u}_i$ is the instantaneous filtered velocity component in the i-direction, and $\overline{p}$ is the instantaneous filtered pressure. The effect of the unresolved subgrid-scales appears in the SGS stress as follows:

$$\tau_{ij} = \overline{u_i u_j} - \overline{u}_i \overline{u}_j$$

(7),

which must be modeled. In this study, τ_{ij} is parameterized by an eddy viscosity assumption of Smagorinsky [1] through the following constitutive relations:

$$\tau_{ij} - (\delta_{ij}/3)\tau_{kk} = -2\nu_{SGS}\overline{S}_{ij}$$

(8),

$$\nu_{SGS} = (C_s f_s \Delta)^2 |\overline{S}|$$

(9),

$$f_s = 1 - \exp(-z^+/25)$$

(10),

$$|\overline{S}| = (2\overline{S}_{ij}\overline{S}_{ij})^{1/2}$$

(11),

$$\overline{S}_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right)$$

(12),

$$\Delta = (h_x h_y h_z)^{1/3}$$

(13),

where δ_{ij} is the Kronecker delta, ν_{SGS} is the eddy viscosity, $\overline{S}_{ij}$ is the resolved strain-rate tensor, C_S (=0.1) is the dimensionless model coefficient (i.e., Smagorinsky constant), which is multiplied by the Van Driest exponential wall damping function f_s in order to account for the near wall effect, Δ is the grid-filter width, which is a characteristic length scale of the largest subgrid-scale eddies, and $|\overline{S}|$ is the magnitude of the resolved strain-rate tensor.

Coordinate system and variable arrangement

The most important factor involved in a successfully accurate simulation of airflow over complex terrain is correctly determining how to specify the topography model as the boundary conditions in the computation. In the present study, we employ a generalized curvilinear collocated grid, where the Cartesian velocity components and pressure are defined at the center of a cell, while the volume flux components multiplied by the Jacobian are defined at the mid-point on their corresponding cell surfaces. The original governing equations in the physical space are transformed to the computational space through a coordinate transformation (see Fig.1 and Fig.2).

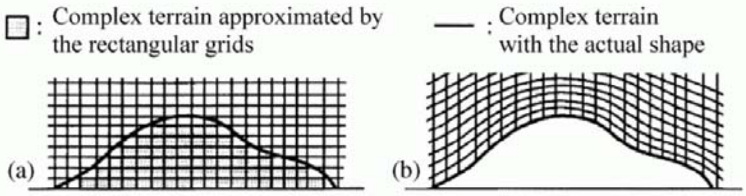

Fig. 1. Coordinate system: (a) cartesian coordinate system; (b) generalized curvilinear coordinate system.

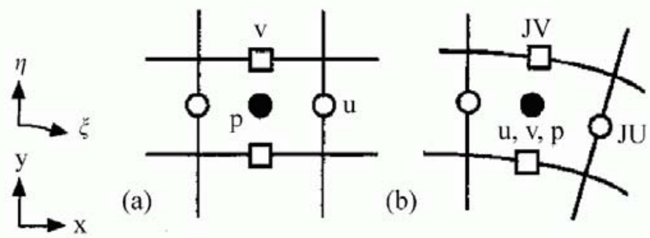

Fig. 2. Variable arrangement: (a) staggered grid; (b) collocated grid.

Numerical method

The coupling algorithm of the velocity and pressure fields is based on a fractional step method [(2)] with the Euler explicit scheme. Therefore, the velocity and pressure fields are integrated by the following procedure. In the first step, the intermediate velocity field is calculated from the momentum equations without the contribution of the pressure gradient. In the next step, the pressure field is computed iteratively by solving the Poisson equation with the SOR (Successive Over Relaxation) method. Finally, the divergence-free velocity at the (n+1) time-step is then obtained by correcting the intermediate velocity field with the computed pressure gradient. As for the spatial discretization in the governing equations, a second-order accurate central difference approximation is used, except for the convective terms. For the convective terms written in non-conservation form, a modified third-order upwind biased scheme [(3)] is used. The weight of the numerical viscosity term is sufficiently small (α=0.5), compared to the Kawamura-Kuwahara scheme (α=3) [(4)] (see Table 1).

Table 1
Characteristics of the RIAM-COMPACT

	Code I	Code II
Coordinate system	Cartesian coordinate system	Generalized curvilinear coordinate system
Variable arrangement	Staggered grid	Collocated grid
Discretization method	Finite-difference method (FDM)	
Coupling algorithm	Fractional step method	
Time advancement method	Euler explicit method	
Poisson equation for pressure	Successive over relaxation (SOR) method	
Convective terms	3rd-order upwind biased scheme based on an interpolation method ($\alpha = 0.5$)	
Other spatial derivative terms	2nd-order accurate central difference approximation	
SGS model	Standard Smagorinsky model with the wall damping function	

References

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